

1. Sketch the graph of each function
  - a.  $y = 4e^{-x}$
  - b.  $y = 2e^x + 5$
  - c.  $y = 5 - 3e^x$
  - d.  $y = \frac{1}{2}e^{-3x} + 1$
  - e.  $y = -2e^{-x} + 5$
  
2. The population of rabbits in a wood is increasing according to the formula  $P = 50 + 10e^{2t}$  where  $P$  is the population of rabbits and  $t$  is the time in years after 2000.
  - a. State the population in the year 2000.
  - b. Predict the population in the year 2030.
  - c. Sketch the graph of  $P$  against  $t$  from 2000 to 2020.
  
3. Sketch the graph of each function stating asymptotes and intercepts
  - a.  $y = \ln(-x)$
  - b.  $y = 2 + 5\ln x$
  - c.  $y = \ln(5 - x)$
  - d.  $y = (\ln x)^2$
  - e.  $y = \ln(5x) + 5$
  
4. Sketch  $h(x) = 2 + \ln(3 - x)$ 
  - a. show the  $x$  and  $y$  intercepts
  - b. state the range and domain of  $h(x)$
  - c. find the inverse function  $h^{-1}(x)$
  
5. Solve the following equations giving exact answers
  - a.  $e^{3x} = 2^2$
  - b.  $2e^{-x} = 5$
  - c.  $3 \ln(x) = 2$
  - d.  $\ln(2x - 1) = 6$
  - e.  $3e^{3x} - 6 = 1$
  - f.  $4e^{2x+1} = 12e^{x+2}$
  - g.  $4 \ln\left(\frac{x}{3}\right) - 1 = 0$
  
6. The functions  $m$  and  $n$  are valid for all real numbers are  $m: x \rightarrow 2x - 1$  and  $n: x \rightarrow 3e^{-2x}$ 
  - a. state the range of  $n$
  - b. sketch  $m^{-1}$  and  $n^{-1}$
  - c. how many roots are there to  $m^{-1}(x) = n^{-1}(x)$
  - d. find  $mn\left(-\frac{1}{4}\right)$
  
7. Solve the following equations giving the solution in terms of  $\ln 3$ 
  - a.  $e^{2x} = 9$
  - b.  $e^{-5x} = \frac{1}{27}$
  - c.  $2e^{3x} - 6 = 0$
  
8. The price of a laptop can be modelled by the equation  $P = 350 + e^{-\frac{t}{2}}$  where  $P$  is price in £s and  $t$  is age in years since purchase.
  - a. Calculate the price of the new laptop.
  - b. Calculate the price after 5 years.
  - c. When will it be worth less than £210?
  - d. Find its price at  $t = \infty$ .
  - e. Sketch  $P$  against  $t$ .
  - f. Is this an appropriate model?
  
9. The function  $h(x) = 20 - 5e^{4x}$   $\{x > 0, x \in \mathbb{R}\}$ 
  - a. state the range of  $h(x)$
  - b. find  $x$  to 2dp when  $h(x) = 14$
  - c. find the exact value of the  $y$ - and  $x$ -intercepts
  - d. find  $h^{-1}(x)$
  - e. sketch  $h^{-1}(x)$  stating its range and domain
  
10. The points  $P$  and  $Q$  lie on the curve  $y = \frac{1}{2}e^{-x}$ . The  $x$ -coordinate of  $P$  is  $\ln(4)$  and the  $y$ -coordinate of  $Q$  is  $e^4$ .
  - a. Find the equation of the line that passes through  $P$  and  $Q$ , using exact values.
  - b. Find the value of the  $y$ -intercept of line through  $P$  and  $Q$
  - c. Calculate the length of  $PQ$ .