1. Show that each equation has roots between the stated values.
a. $x^3 - 3x^2 - 3x + 5$ between 3.4 and 3.6
b. $2x^3 + x^2 - 4x - 2$ between 1.41 and 1.44
c. $e^x - \sin(x)$ between 0.2 and 0.4
d. $3(\cos(x))^2$ +5sin(x) between -0.6 and -0.4
e. $2e^x + 2x - x$ between 1.0 and 1.4
f. $\sin(3x) + \cos(2x)$ between -0.4 and-0.2

2. Given the equation
$y = x^3 - x^2 - 5x + 4$
a. show that it has a root between 2.40 and 2.39
b. sketch the graph showing intercepts and turning points
c. show that the equation for roots can be written as $x = \sqrt{x + 5 - \frac{4}{x}}$
d. show that the equation for roots can also be written as $x = \frac{1}{5}(x^3 - x^2 + 4)$
e. use both the equations (c and d) to find the root between 2.40 and 2.39 to 3dp

3. Given the equation
$y = 3x^3 + 2x^2 - x + 4$
a. show that it has a root between -1.4 and -1.5
b. sketch the graph showing intercepts and turning points
c. show that the equation for roots can be written as $x = \sqrt{\frac{1}{3} - \frac{2}{3}x - \frac{4}{3x}}$
d. use the equation to find a root to 3dp

4. Sketch the graph of each function stating asymptotes and intercepts
a. $y = \ln(-x)$
b. $y = 2 + 5lnx$
c. $y = \ln(5 - x)$
d. $y = (lnx)^2$
e. $y = \ln(5x) + 5$

5. Sketch h(x) = 2 + ln (3 – x)
a. show the x and y intercepts
b. state the range and domain of h(x)
c. find the inverse function $h^{-1}(x)$

6. Solve the following equations giving exact answers
a. $e^{3x} = 2^2$
b. $2e^{-x} = 5$
c. $3\,ln(x) = 2$
d. $ln(2x - 1) = 6$
e. $3e^{3x} - 6 = 1$
f. $4e^{2x+1} = 12e^{x+2}$
g. $4\,ln\left(\frac{x}{3}\right) - 1 = 0$

7. The functions m and n are valid for all real numbers are $m: x \to 2x - 1$ and $n: x \to 3e^{-2x}$
a. state the range of n
b. sketch m^{-1} and n^{-1}
c. how many roots are there to $m^{-1}(x) = n^{-1}(x)$
d. find $mn(-\frac{1}{4})$

8. Solve the following equations giving the solution in terms of ln3
a. $e^{2x} = 9$
b. $e^{-5x} = \frac{1}{27}$
c. $2e^{3x} - 6 = 0$

9. The price of a laptop can be modelled by the equation
$P = 350 + e^{-\frac{t}{2}}$
where P is price in £s and t is age in years since purchase.
a. Calculate the price of the new laptop.
b. Calculate the price after 5 years.
c. When will it be worth less than £210?
d. Find its price at t = ∞.
e. Sketch P against t.
f. Is this an appropriate model?

10. The function $h(x) = 20 - 5e^{4x}$ $\{x > 0, x \in \mathbb{R}\}$
a. state the range of $h(x)$
b. find x to 2dp when $h(x)$ = 14
c. find the exact value of the y- and x-intercepts
d. find $h^{-1}(x)$

e. sketch $h^{-1}(x)$ stating its range and domain

11. The points P and Q lie on the curve $y = \frac{1}{2}e^{-x}$. The x-coordinate of P is ln(4) and the y-coordinate of Q is e^4.

a. Find the equation of the line that passes through P and Q, using exact values.
b. Find the value of the y-intercept of line through P and Q
c. Calculate the length of PQ.